Math 106: Homework 2

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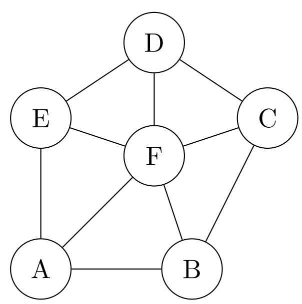
Due: Monday, September 29, 2025

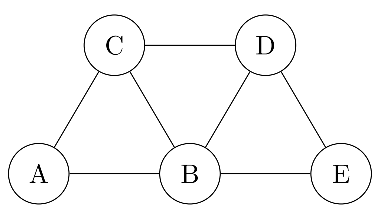
Instructions for Homework Turn In

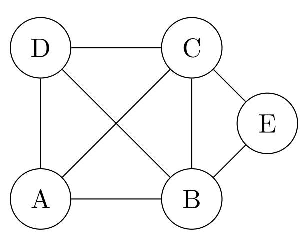
1. You are required to answer **all questions thoroughly** and strictly **in the order presented**.
2. **Show all work clearly** and provide complete, logical reasoning for each step.
3. Handwritten work should be neat and organized. Illegible or disorganized work may not be graded.
4. Include **drawings and diagrams** where applicable. Neatly hand-drawn graphs are acceptable if clear and legible.
5. Write all descriptive or explanatory responses in **complete, grammatically correct sentences**.
6. Write each answer on a separate sheet of paper. DO NOT use this document to write your answers. Use this PDF only for referring to a particular question you are answering.
7. Failure to comply with any of these instructions **may result in deduction of points**.

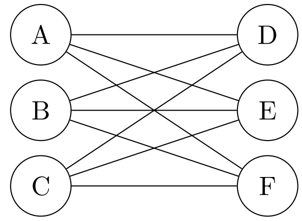
Set 1: Chromatic Number

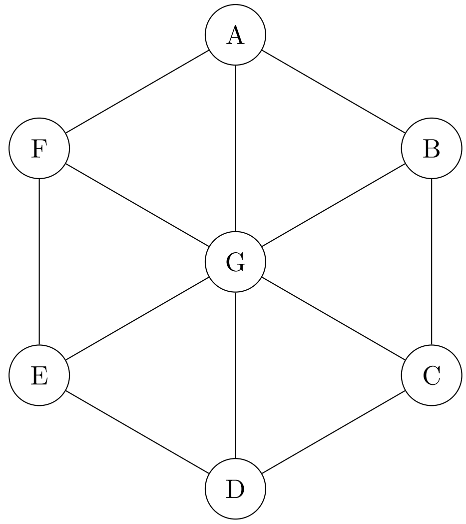
1. Define the chromatic number of a graph.
2. Find the chromatic number of the following graphs and provide a valid coloring:

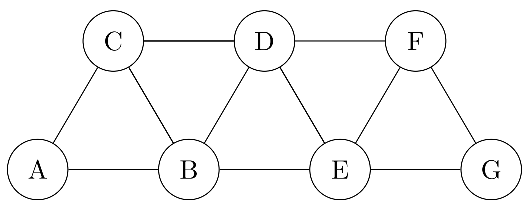


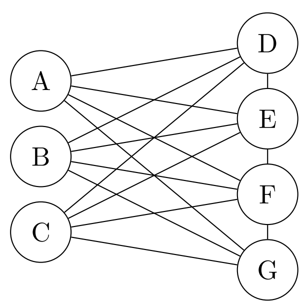










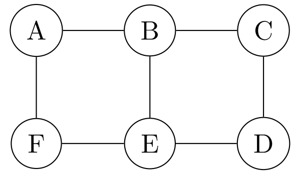


3. Prove or disprove: The chromatic number of any tree is 2.

Set 2: Hamiltonian Paths, TSP, and Algorithms

1. What is the difference between an Euler circuit and a Hamiltonian circuit? Give an example of a graph that has one but not the other.

2. Determine whether the following graph has a Hamiltonian circuit:



3. **Traveling Salesman Problem (TSP) – Nearest Neighbor Approximation**

   A salesman must visit each of the following cities exactly once and return to the starting point. The distances (in miles) between the cities are given below:

From/To	A	B	C	D
A	–	10	15	20
B	10	–	35	25
C	15	35	–	30
D	20	25	30	–

   a) Starting from city A, use the **Nearest Neighbor Algorithm** to find an approximate solution to the TSP.

   b) List the order in which the cities are visited.

   c) What is the total distance of the tour?

   d) Briefly explain why the solution may not be optimal.

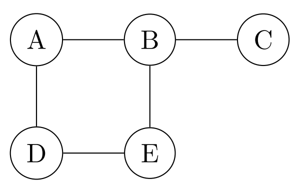
4. Use **the Sorted Edges Algorithm** to solve the same TSP problem in question 3.

5. Define a complete graph. How many edges are in a complete graph with $n = 6$ vertices?

Set 3: Trees, Spanning Trees, and Kruskal's Algorithm

1. Define a tree. List at least two properties that distinguish a tree from other types of graphs.

2. Draw all spanning trees of the following graph:



3. **Use Kruskal's Algorithm** to find the **Minimum Spanning Tree (MST)** of the following weighted graph.

Edge	Weight
A–B	3
A–C	1
B–C	7
B–D	5
C–D	4
C–E	6
D–E	2

a) List the edges in the order they are selected by Kruskal's Algorithm.
b) Draw the resulting Minimum Spanning Tree (MST).
c) What is the total weight of the MST?
d) Why would Kruskal's Algorithm not select some edges, even if they connect important nodes?

4. Given the following **weighted adjacency matrix**, apply **Kruskal's Algorithm** to find the Minimum Spanning Tree (MST).

	A	B	C	D	E
A	0	2	3	0	0
B	2	0	1	4	0
C	3	1	0	5	6
D	0	4	5	0	2
E	0	0	6	2	0

a) Convert the matrix into a list of edges with their weights.
b) Use Kruskal's Algorithm to find the MST. Show the steps and the order of edge selection.
c) What is the total weight of the MST?

5. Consider the following task times:

Task	**Time (in units)**
A	3
B	2
C	4
D	2
E	3
F	1

The tasks have the following precedence relations:

- Task A must be completed before tasks B and C can begin.
- Task B must be completed before task D.
- Task C must be completed before tasks D and E.
- Task D and E must be completed before task F.

a) Draw the task dependency graph (as a directed acyclic graph).
b) Perform a topological sort of the tasks.
c) Determine the earliest start and finish times for each task.
d) Identify the **critical path** and total project completion time.

6. A delivery truck must visit four cities: A, B, C, and D. The distance between each pair of cities is given:

- A–B: 12, A–C: 10, A–D: 15
- B–C: 9, B–D: 11
- C–D: 14

a) Use the **Nearest Neighbor Algorithm**, starting at city A, to determine the approximate tour.

b) What is the total distance traveled?

c) Does the tour end where it started? If not, complete the cycle.

7. Consider a set of cities $\{A, B, C, D, E\}$ with the following symmetric distance matrix:

	A	B	C	D	E
A	–	4	8	10	7
B	4	–	6	5	9
C	8	6	–	3	2
D	10	5	3	–	4
E	7	9	2	4	–

a) Apply the Nearest Neighbor Algorithm starting from city A.

b) Apply the same algorithm starting from city C.

c) Compare the total distances. Which starting point gives a better tour?

Set 4: Prove or Disprove

1. Let G be a connected graph with 10 vertices and 15 edges. Prove or disprove: G must contain at least one cycle.

2. A graph has 12 vertices, and each vertex has degree 5. Is such a graph possible? Justify your answer.

3. Consider a graph G with an adjacency matrix such that all off-diagonal entries are either 0 or 1, and all rows sum to 3. Prove that the graph is 3-regular graph[1] and determine if it must be connected.

4. Given a weighted complete graph with 6 vertices and distinct edge weights, explain why Kruskal's Algorithm always returns a unique Minimum Spanning Tree.

5. Draw a non-Hamiltonian planar graph with all vertices of degree at least 3. Justify why it is not Hamiltonian.

6. The chromatic number of a graph G is 4. Does it necessarily contain a subgraph similar to K_4[2] ? Prove or provide a counterexample.

7. A graph G has an Euler circuit but does not have a Hamiltonian circuit. Construct such a graph with at least 6 vertices and explain why it satisfies these properties.

8. A salesman must visit 7 cities where the distances between each pair of cities are unique. Is the optimal solution to the TSP (Traveling Salesman Problem) always unique in this case? Justify your answer.

[1] A graph is called *3-regular* if every vertex has degree exactly 3, i.e., each vertex is connected to exactly 3 edges.
[2] K_4 is the complete graph on 4 vertices, meaning every pair of distinct vertices is connected by an edge.