Math 106: Homework 3

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Due: Monday, October 6, 2025

Instructions for Homework Turn In

1. You are required to answer **all questions thoroughly** and strictly **in the order presented**.
2. **Show all work clearly** and provide complete, logical reasoning for each step.
3. Handwritten work should be neat and organized. Illegible or disorganized work may not be graded.
4. Include **drawings and diagrams** where applicable. Neatly hand-drawn graphs are acceptable if clear and legible.
5. Write all descriptive or explanatory responses in **complete, grammatically correct sentences**.
6. Write each answer on a separate sheet of paper. DO NOT use this document to write your answers. Use this PDF only for referring to a particular question you are answering.
7. Failure to comply with any of these instructions **may result in deduction of points**.

Problem 1

Maximize

$$Z = 7x + 11y$$

subject to

$$\begin{cases} 3x + 5y \leq 26, \\ 5x + 3y \leq 30, \\ x \geq 0, \quad y \geq 0. \end{cases}$$

Problem 2

Minimize

$$Z = 4x + 6y$$

subject to

$$\begin{cases} x + 2y \geq 80, \\ 3x + y \geq 75, \\ x \geq 0, \ y \geq 0. \end{cases}$$

Problem 3

Consider the linear objective function

$$Z = px + qy,$$

subject to the system of inequalities:

$$\begin{cases} 2x + y \leq 10, \\ x + 3y \leq 15, \\ x \geq 0, \quad y \geq 0. \end{cases}$$

The feasible region determined by these constraints has the corner points (vertices):

$$(0, 0), \quad (5, 0), \quad (3, 4), \quad (0, 5).$$

Assume $p > 0$ and $q > 0$.

Question: Determine the condition on p and q such that the maximum value of

$$Z = px + qy$$

is attained **simultaneously** at the two vertices $(3, 4)$ and $(0, 5)$.

Problem 4

A company manufactures two types of dresses, C and D. The resource usage per dress and the total availability per day are given in the table below:

Resource	Dress $C\,(x)$	Dress $D\,(y)$	Maximum availability per day
Raw material	5	4	60
Labour	5	3	50

If the profit per unit of dress C is \$50 and per unit of dress D is \$100, then:

(a) Formulate the linear programming problem clearly (i.e., write the objective function and all constraints).

(b) Determine the coordinates of the vertices (corner points) of the feasible region.

(c) Evaluate the profit P at each vertex and hence find the optimal production plan (x, y) that maximizes the profit.

(d) State the maximum profit.