Introduction to Mathematical Modeling

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1. What is a Maximization Problem?

In everyday life, we often try to get the **best outcome** with limited resources.

Examples:

- Running a business: maximize profit with limited time and materials.
- Shopping: get the most value for your money.
- Scheduling: make the most of limited hours in a day.

In math, we call this a **maximization problem**, where the goal is to make something (like profit or efficiency) as large as possible, *without breaking any rules.*

2. Understanding x- and y-Intercepts

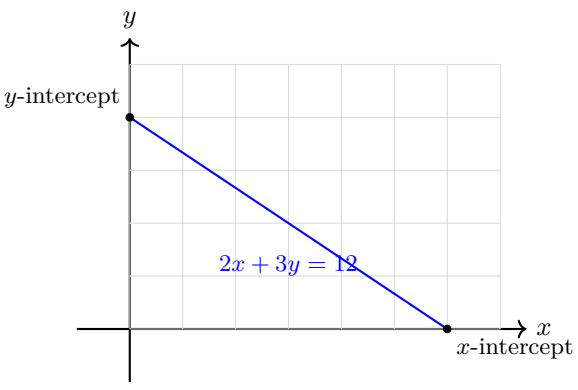
x-intercept: Where the line crosses the x-axis. (Set $y = 0$)
y-intercept: Where the line crosses the y-axis. (Set $x = 0$)

Example:

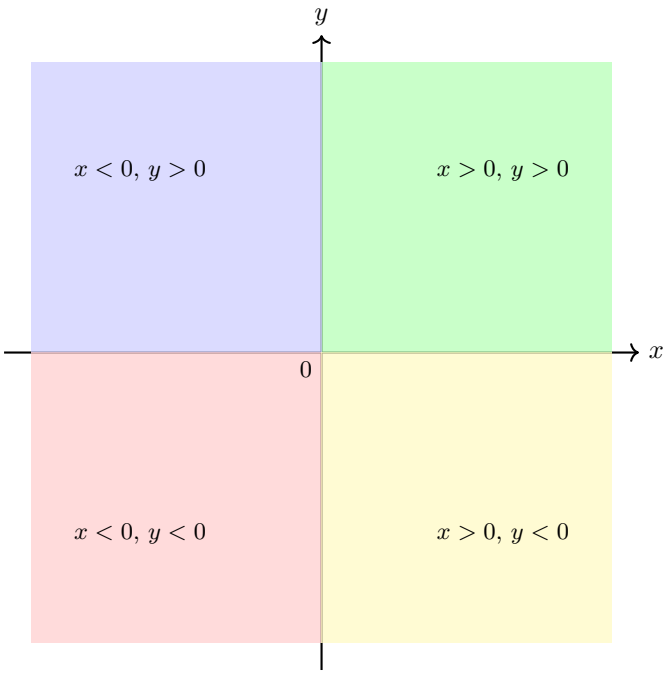
$$2x + 3y = 12$$

- To find the **x-intercept**, set $y = 0$:
  $2x = 12 \Rightarrow x = 6 \Rightarrow (6, 0)$
- To find the **y-intercept**, set $x = 0$:
  $3y = 12 \Rightarrow y = 4 \Rightarrow (0, 4)$

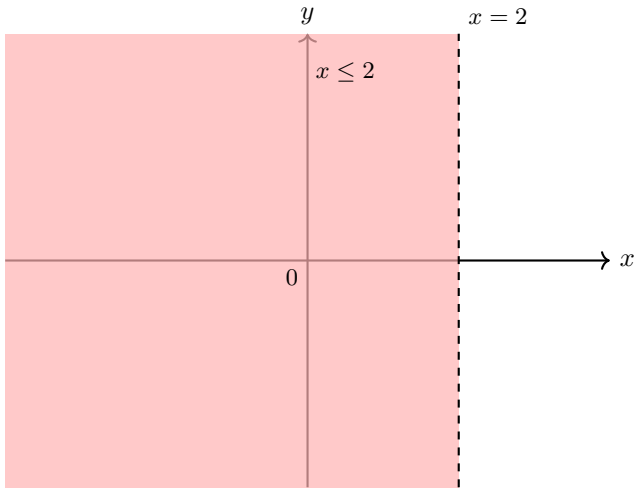
We use intercepts to quickly draw lines when graphing constraints.



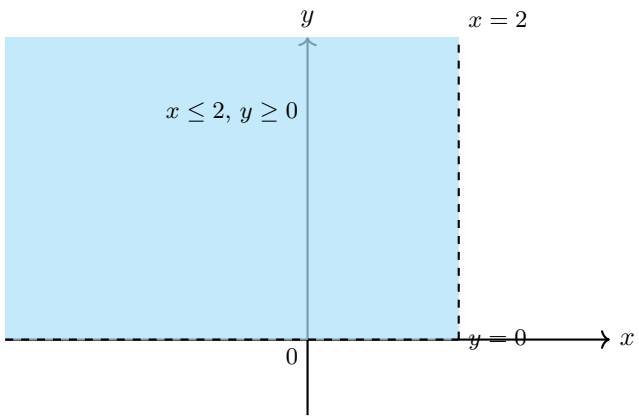
3. Understanding Quadrants in the XY-Plane



Plot $x \leq 2$



Plot $x \leq 2, y \geq 0$



4. What Are Constraints?

Constraints are the **rules or limits** in a problem. They tell us what is allowed and what is not.

Example Constraints:

$$\begin{aligned} x + y &\leq 6 \quad \text{(Cant make more than 6 items)} \\ x &\geq 0 \quad \text{(Cant make negative items)} \\ y &\geq 0 \quad \text{(Cant make negative items)} \end{aligned}$$

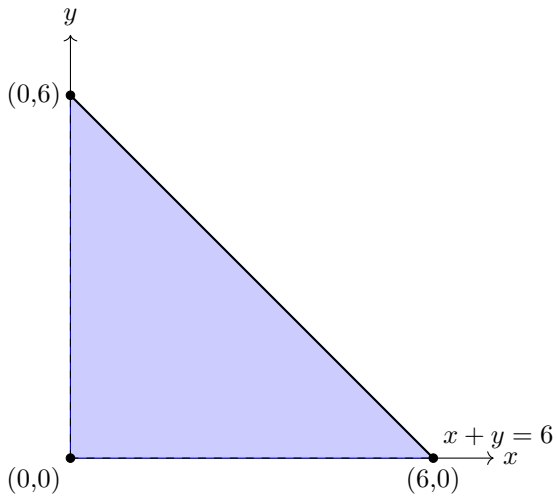
We graph each constraint using its intercepts.

5. Graphing Constraints Using Intercepts

Lets graph: $x + y \leq 6$

Find Intercepts:

- x-intercept: $y = 0 \Rightarrow x = 6$
- y-intercept: $x = 0 \Rightarrow y = 6$



The shaded area is where all rules are followed. This is the **feasible region**.

Practice Problems Set 1

1. Where do the lines $3x + 5y = 26$ and $2x + 3y = 16$ intersect?
   (a) At the point $(2, 4)$
   (b) At the point $(6, 2)$
   (c) At the point $(4, 2)$

2. The x-coordinate and y-coordinate of the point where the line $x = 22$ intersects the line $5y - 3x = 29$ are _____ and _____, respectively.

3. The y-coordinate of the point whose x-value is 3 on the line $3x + 2y = 12$ is
   (a) $\frac{3}{2}$
   (b) 3
   (c) 2

4. The lines $x + 3y = 12$ and $y = 3$ intersect at the point with x-coordinate _____ and y-coordinate _____.

5. The two lines $2x + 3y = 12$ and $6x + 9y = 7$
   (a) intersect at $(3, 2)$.
   (b) intersect at $(23, 22)$.
   (c) are parallel.

6. The x-coordinate and y-coordinate of the points where the line $7x + 2y = 28$ crosses the x-axis and y-axis, respectively, are _____ and _____.

7. Which of these points lie in the region defined by

$$4x + 3y \leq 24, \quad x \geq 0, \quad y \geq 0?$$

(a) Points $(2, 5)$ and $(3, 4)$
(b) Points $(5, 2)$ and $(3, 4)$
(c) Points $(5, 2)$ and $(2, 5)$

Practice Problems Set 2

1. Find the coordinates of all points where the lines

$$x + y = 8, \quad y = 5, \quad \text{and} \quad x = 1$$

intersect.

2. Find the x- and y-intercepts of the line

$$6y + 5x = 30.$$

3. Graph the lines $x + y = 10$, $x = 3$, and $y = 4$ on the same set of axes. For each pair of these three lines, find the x- and y-coordinates where the lines intersect.

4. Using intercepts, the points where the lines cross the axes, graph each line:
   (a) $7x + 6y = 42$
   (b) $x = 2$
   (c) $y = 5$

5. Using intercepts, graph each line:
   (a) $2x + 3y = 12$
   (b) $3x + 5y = 30$
   (c) $4x + 3y = 24$

6. (a) Graph the lines $x = 2$, $y = 3$, $x = 4$, and $y = 7$ on the same set of axes.
   (b) The points where the four lines meet form what kind of geometric figure (shape)?

7. Graph both lines on the same axes. Put a dot where the lines intersect. Use algebra to find the x- and y-coordinates of the point of intersection.
   (a) $4x + 6y = 18$ and $x = 0$
   (b) $5x + 3y = 45$ and $y = 25$
   (c) $5x + 3y = 45$ and $x = 3$

Graphing Inequalities

Graph the line and half-plane corresponding to the inequality, a typical constraint from a mixture problem:

(a) $x \leq 6$

(b) $y \geq 4$

(c) $5x + 3y \leq 15$

(d) $4x + 5y \leq 30$

Graph the line and half-plane corresponding to the inequality, another set of typical mixture problem constraints:

(a) $x \leq 2$

(b) $y \geq 8$

(c) $3x + 2y \leq 18$

(d) $7x + 2y \leq 42$

End of Lecture #7