Math 106: Homework 5

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Due: Monday, November 3, 2025

Instructions for Homework Turn In

1. Answer all questions in the order presented and show all work clearly.
2. Use proper mathematical notation and complete sentences for explanations.
3. If you use a calculator or computer, label each computation carefully.
4. Write neatly; unreadable work may not receive credit.
5. You may include graphs or tables if they help your explanation.

Problem 1: UPC Codes and Check Digits

A **UPC (Universal Product Code)** uses a *check digit* to detect scanning or data-entry errors.
For a UPC code:

$$a_1a_2a_3a_4a_5a_6a_7a_8a_9a_{10}a_{11}a_{12},$$

the last digit a_{12} is the check digit chosen so that

$$3(a_1 + a_3 + a_5 + a_7 + a_9 + a_{11}) + (a_2 + a_4 + a_6 + a_8 + a_{10})$$

is divisible by 10.

(a) Compute the missing check digit for the code

$$0\ 6\ 4\ 1\ 4\ 4\ 2\ 8\ 2\ 6\ 3\ ?$$

(b) Verify that your completed UPC code satisfies the divisibility rule.

(c) Explain briefly how the weighting of digits (by 3 or 1) helps detect single-digit and transposition errors.

Problem 2: Modular Arithmetic and Error Detection

When using check digits, we often rely on **modular arithmetic** to express "remainder" relationships. For integers a, b, and $m \geq 2$, we write

$$a \equiv b \pmod{m}$$

if m divides $a - b$.

(a) Determine whether each statement is true or false:

- $25 \equiv 1 \pmod{6}$
- $100 \equiv 20 \pmod{10}$
- $52 \equiv 0 \pmod{13}$
- $75 \equiv 7 \pmod{5}$

(b) Compute the following:

$$34 \bmod 5 = \_\_\_\_\_$$
$$78 \bmod 11 = \_\_\_\_\_$$
$$13 \bmod 15 = \_\_\_\_\_$$
$$12 \bmod 2 = \_\_\_\_\_$$

(c) Explain in your own words how modular arithmetic is used in UPC check digit systems.

Problem 3: Encoding Data in Illinois Driver's License Numbers

The last 5 digits of Illinois driver's license numbers encode the driver's birth year and gender as follows:

- For a **man**:
  - The last two digits of the birth year (e.g., 67 for 1967),
  - followed by the day of the year assuming every month has 31 days.
- For a **woman**, 600 is added to the number calculated above.

Formula for Day of Year:

$$\text{Day of year} = 31 \times (\text{month} - 1) + d$$

(a) Find the last 5 digits for a man born on February 12, 1967.

(b) Interpret what the last 5 digits `10642` tell you about the person.

(c) Interpret what the last 5 digits `90373` tell you about the person.

Problem 4: Encoding Data in a Hypothetical State's Driver's License Numbers

In another state, the last 5 digits of a driver's license encode the birth year and gender as follows:

- For a **man**: the last two digits of the year followed by the day of the year (assuming every month has 30 days),
- For a **woman**: 500 is added to the number calculated above.

Formula for Day of Year:

$$\text{Day of year} = 30 \times (\text{month} - 1) + d$$

(a) Find the last 5 digits for a man born on April 20, 1983.

(b) Interpret what the last 5 digits `84525` tell you about the person.

(c) Interpret what the last 5 digits `79270` tell you about the person.

Problem 5: Encoding Birth Day in ID Numbers with 35-Day Months

In a hypothetical system, the last three digits of a man's ID number represent the birth day of the year, assuming each month has 35 days. For a woman, 500 is added to this number.

(a) What are the last three digits of a man's ID number if he was born on October 8th?

(b) What do you know about a person if the last three digits of the ID number are `503`?

(c) What do you know about a person if the last three digits of the ID number are `420`?