Introduction to Mathematical Modeling

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Converting Decimal to Binary and Binary to Decimal

Introduction

In digital systems, numbers are represented in the **binary system**, which uses only two digits: 0 and 1. Humans commonly use the **decimal system**, based on ten digits (0–9). Understanding how to convert between these systems is fundamental in computer science, electronics, and data modeling.

Positional Number Systems

Every number system is based on a **base** or **radix**:

- Decimal system: base 10
- Binary system: base 2

In a positional system, each digit represents a power of the base:

$$(345)_{10} = 3 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$$

$$(1011)_2 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

Converting Decimal to Binary

Definition 1 (Division–Remainder Method)**.** *To convert a decimal number to binary:*

1. *Divide the decimal number by* 2*.*
2. *Record the remainder (*0 *or* 1*).*
3. *Divide the quotient again by* 2*.*
4. *Continue dividing until the quotient becomes* 0*.*
5. *Read the remainders from* ***bottom to top****.*

Example: Convert 45_{10} to binary.

Step	Division by 2	Remainder
1	$45 \div 2 = 22$	1
2	$22 \div 2 = 11$	0
3	$11 \div 2 = 5$	1
4	$5 \div 2 = 2$	1
5	$2 \div 2 = 1$	0
6	$1 \div 2 = 0$	1

$$(45)_{10} = (101101)_2$$

Converting Binary to Decimal

Definition 2. *To convert a binary number to decimal:*

1. *Write the binary number.*
2. *Multiply each bit by 2^n, where n is the position index from right (starting at 0).*
3. *Sum all results.*

Example:

$$(101101)_2 = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 45$$

Thus, $(101101)_2 = (45)_{10}$.

Practice Problems

1. Convert 27_{10} to binary.
2. Convert 110101_2 to decimal.
3. Verify that converting one way and back yields the original number.

The Caesar Cipher

The **Caesar Cipher** is one of the simplest and earliest encryption methods. It shifts each letter by a fixed number of positions in the alphabet.

Encryption

Definition 3. *Let plaintext $P = p_1, p_2, \ldots, p_n$. Each letter is replaced by:*

$$C_i = (P_i + k) \bmod 26$$

*where k is the shift (**key**).*

Example: Encrypt `HELLO` with $k = 3$:

Letter	Value	Encrypted
H	7	K
E	4	H
L	11	O
L	11	O
O	14	R

`HELLO` → `KHOOR`

Decryption

Shift in the opposite direction:

$$P_i = (C_i - k) \bmod 26$$

`KHOOR` → `HELLO`

The Decimation Cipher

Introduction

The **Decimation Cipher** multiplies each letter's numeric value by a key k, using modular arithmetic. It requires k to be an **odd number not equal to 13** to ensure invertibility.

Encryption Steps

1. Assign $A = 0, B = 1, \ldots, Z = 25$.
2. Choose a valid key k.
3. Compute $E(x) = (k \times x) \bmod 26$.

Example: Encrypt `CODE` with $k = 5$:

$$C(2) \to K,\ O(14) \to S,\ D(3) \to P,\ E(4) \to U$$

`CODE` → `KSPU`

Why Not 13?

Keys that are even or equal to 13 have no modular inverse mod 26, making decryption impossible.

Decryption

Use the inverse key j, such that $(k \times j) \bmod 26 = 1$. Then compute $D(x) = (j \times x) \bmod 26$.

Example: If $k = 5$, its inverse is $j = 21$.

$$\texttt{KSPU} \rightarrow \texttt{CODE}$$

Practice

1. Encrypt `MATH` using $k = 7$.
2. Decrypt a message encoded with $k = 9$.
3. Explain why $k = 13$ or any even number fails.

Example: Encrypting `THANKS` with $k = 11$

$$E(i) = (11 \times i) \bmod 26$$

Letter	Value	$11 \times i \bmod 26$	Encrypted
T	19	1	B
H	7	25	Z
A	0	0	A
N	13	13	N
K	10	6	G
S	18	16	Q

$$\boxed{\texttt{THANKS} \longrightarrow \texttt{BZANGQ}}$$

Example: Decrypting with $k = 21$, inverse $j = 5$

$$D(x) = (5 \times x) \bmod 26$$

Cipher	Value	$5 \times x \bmod 26$	Plain
Q	16	2	C
R	17	7	H
G	6	4	E
S	18	12	M
M	12	8	I
O	14	18	S
J	9	19	T
T	19	17	R
K	10	24	Y

$\boxed{\texttt{Q R G S M O J T K} \longrightarrow \texttt{CHEMISTRY}}$

End of Lecture #12