MATH 108: Elementary Probability and Statistics

Ramapo College of New Jersey

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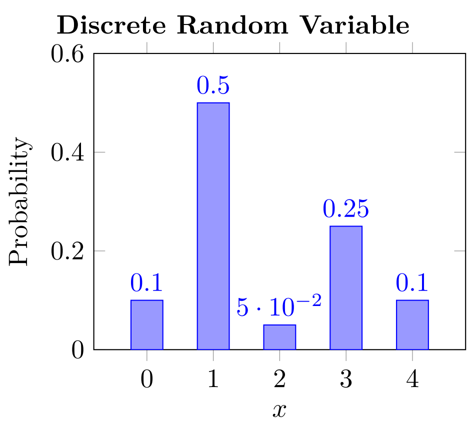
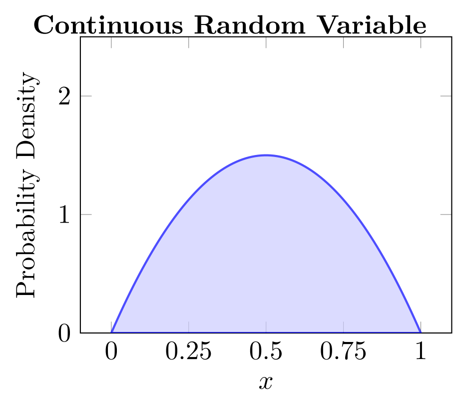
Discrete vs. Continuous Random Variables

Discrete Random Variable: A random variable that can take on a finite or countably infinite set of distinct values. Examples include the number of students in a classroom or the outcome of rolling a die (e.g., $0, 1, 2, 3, \ldots$).

Continuous Random Variable: A random variable that can take on any value within a given interval on the real number line. The possible values are uncountably infinite, such as any real number between 0 and 1 (e.g., $x \in [0, 1]$).





Examples:

- Discrete: Number of students in a class, number of calls received in an hour.
- Continuous: Height of students, temperature, time to run a mile.

Identifying Discrete Probability Distributions

A discrete probability distribution must satisfy:

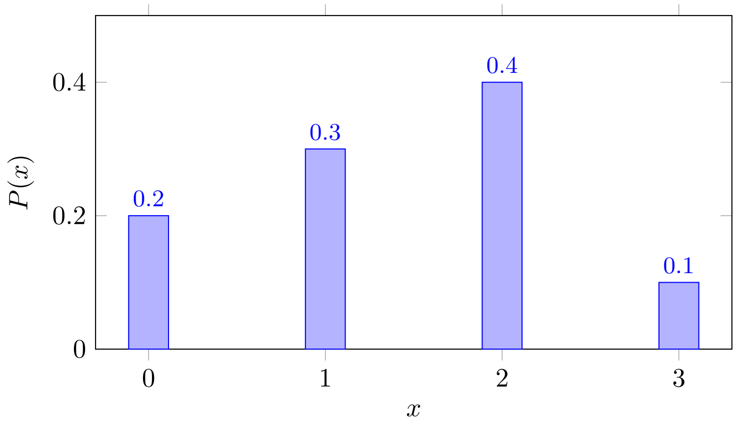
- Each probability $P(x)$ is between 0 and 1: $0 \leq P(x) \leq 1$
- The sum of all probabilities is 1: $\sum P(x) = 1$

Graphing Discrete Probability Distributions

We use bar graphs (probability histograms) where the x-axis represents the variable x and the y-axis represents the probability $P(x)$.

Example:

x	0	1	2	3
$P(x)$	0.2	0.3	0.4	0.1



Example 1

x	0	1	2	3	4
$P(x)$	0.10	0.50	0.05	0.25	0.10

Step 1: Is the variable x discrete?

The values of x are:

$$x = 0, 1, 2, 3, 4$$

These are all whole numbers (integers), so the variable x is **discrete**.

Step 2: Does it meet the conditions for a discrete probability distribution?

Condition 1: Each probability $P(x)$ satisfies $0 \leq P(x) \leq 1$

$$\begin{aligned} P(0) &= 0.10 \\ P(1) &= 0.50 \\ P(2) &= 0.05 \\ P(3) &= 0.25 \\ P(4) &= 0.10 \end{aligned}$$

All values are between 0 and 1.

Condition 2: The sum of all probabilities must equal 1

$$\sum P(x) = 0.10 + 0.50 + 0.05 + 0.25 + 0.10 = 1.00$$

This condition is also satisfied.

Problem 15

Given:

x	$P(x)$
3	0.4
4	?
5	0.1
6	0.2

Let the missing probability be $P(4) = p$.
Since the sum of all probabilities must equal 1:

$$0.4 + p + 0.1 + 0.2 = 1$$

$$p = 1 - (0.4 + 0.1 + 0.2) = 1 - 0.7 = 0.3$$

Answer: $\boxed{P(4) = 0.3}$

Problem 16

Given:

x	$P(x)$
0	0.30
1	0.15
2	?
3	0.20
4	0.15
5	0.05

Let the missing probability be $P(2) = p$.
Using the rule that the total must be 1:

$$0.30 + 0.15 + p + 0.20 + 0.15 + 0.05 = 1$$

$$p = 1 - (0.30 + 0.15 + 0.20 + 0.15 + 0.05) = 1 - 0.85 = 0.15$$

Answer: $\boxed{P(2) = 0.15}$

Computing and Interpreting the Mean μ of a Discrete Random Variable

The **mean** or **expected value** is given by:

$$\mu = E(x) = \sum x \cdot P(x)$$

Example:

x	$P(x)$
1	0.2
2	0.5
3	0.3

$$\mu = (1)(0.2) + (2)(0.5) + (3)(0.3) = 0.2 + 1.0 + 0.9 = 2.1$$

Interpretation: The average outcome over many trials is expected to be 2.1.

Interpreting the Mean as an Expected Value

The **expected value** represents the long-run average result of a probability experiment.
In real-life applications, it helps in:

- Making business decisions
- Evaluating risk
- Predicting outcomes in gambling, insurance, etc.

Notation: $E(X) = \mu = \sum x \cdot P(x)$

Computing the Standard Deviation σ

Standard Deviation: Measures how much the values of x deviate from the mean.

$$\sigma = \sqrt{\sum (x-\mu)^2 \cdot P(x)}$$

Example (using previous values where $\mu = 2.1$):

$$\begin{aligned}\sigma &= \sqrt{(1-2.1)^2(0.2) + (2-2.1)^2(0.5) + (3-2.1)^2(0.3)} \\ &= \sqrt{(1.21)(0.2) + (0.01)(0.5) + (0.81)(0.3)} \\ &= \sqrt{0.242 + 0.005 + 0.243} = \sqrt{0.49} = 0.7\end{aligned}$$

Interpretation: The values typically deviate by about 0.7 from the mean of 2.1.

Example:

The random variable X represents the number of marriages an individual (age 15 or older) has been involved in. The probability distribution is:

x	$P(x)$
0	0.272
1	0.575
2	0.121
3	0.027
4	0.004
5	0.001

(a) Verify that this is a discrete probability distribution

To be valid:

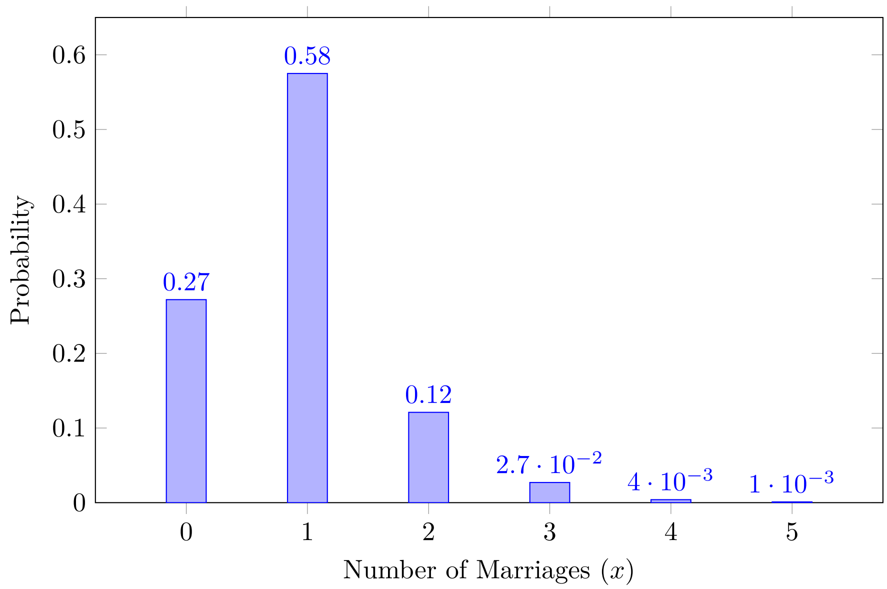
- All $P(x)$ must be between 0 and 1.
- The sum of all probabilities must equal 1:

$$\sum P(x) = 0.272 + 0.575 + 0.121 + 0.027 + 0.004 + 0.001 = 1.000$$

Thus, this is a valid discrete probability distribution.

(b) Graph and Shape of the Distribution

Graph:



Shape: The distribution is **right-skewed (positively skewed)**, as the majority of the probabilities are concentrated at lower values of x (0 or 1), with rapidly decreasing probabilities for higher x.

(c) Mean of the Random Variable X

The mean or expected value is given by:

$$\mu = E(X) = \sum[x \cdot P(x)] = 0(0.272) + 1(0.575) + 2(0.121) + 3(0.027) + 4(0.004) + 5(0.001)$$

$$\mu = 0 + 0.575 + 0.242 + 0.081 + 0.016 + 0.005 = \boxed{0.919}$$

Interpretation: On average, an individual aged 15 or older has been involved in approximately 0.919 marriages.

(d) Standard Deviation of the Random Variable X

First, compute $E(X^2)$:

$$E(X^2) = \sum[x^2 \cdot P(x)] = 0^2(0.272) + 1^2(0.575) + 2^2(0.121) + 3^2(0.027) + 4^2(0.004) + 5^2(0.001)$$
$$= 0 + 0.575 + 0.484 + 0.243 + 0.064 + 0.025 = 1.391$$

Now compute the standard deviation:

$$\sigma = \sqrt{E(X^2) - \mu^2} = \sqrt{1.391 - (0.919)^2} = \sqrt{1.391 - 0.844} = \sqrt{0.547} = \boxed{0.739}$$

(e) Probability of Two Marriages

$$P(X = 2) = \boxed{0.121}$$

(f) Probability of At Least Two Marriages

$$P(X \geq 2) = P(2) + P(3) + P(4) + P(5) = 0.121 + 0.027 + 0.004 + 0.001 = \boxed{0.153}$$

Problem 20: Waiting in Line

A Wendy's manager performed a study to determine a probability distribution for the number of people, X, waiting in line during lunch. The results are shown below:

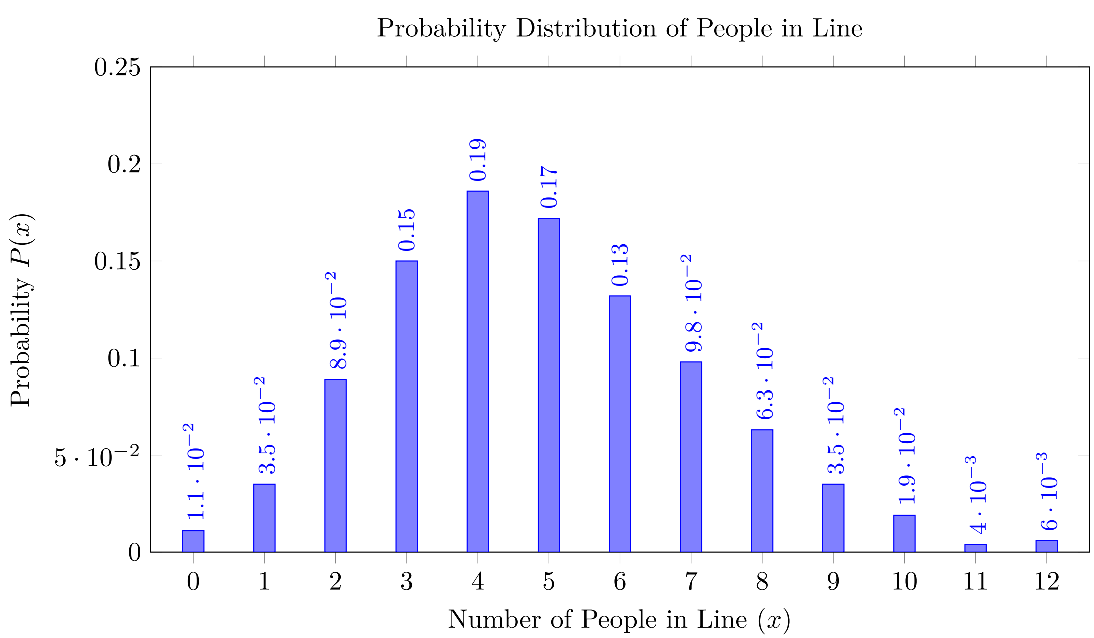
x	$P(x)$
0	0.011
1	0.035
2	0.089
3	0.150
4	0.186
5	0.172
6	0.132
7	0.098
8	0.063
9	0.035
10	0.019
11	0.004
12	0.006

(a) **Verify that this is a discrete probability distribution.**
Solution: Each value of $P(x)$ is between 0 and 1. Also,

$$\sum P(x) = 1.000,$$

so this is a valid discrete probability distribution.

(b) **Draw a graph of the probability distribution. Describe its shape.**
Solution:



The distribution is roughly symmetric with a slight right skew. The peak occurs at $x = 4$.

(c) **Compute and interpret the mean of the random variable X.**
Solution: Using the formula $\mu = \sum x \cdot P(x)$,

$$\mu \approx 4.87.$$

Interpretation: On average, about 4.87 people are waiting in line during lunch.

(d) **Compute the standard deviation of the random variable X.**
Solution: Using

$$\sigma = \sqrt{\sum x^2 P(x) - \mu^2},$$

we find

$$\sigma \approx 1.99.$$

(e) **What is the probability that eight people are waiting in line for lunch?**
Solution:

$$P(8) = 0.063.$$

(f) **What is the probability that 10 or more people are waiting in line? Would this be unusual?**
Solution:

$$P(X \geq 10) = P(10) + P(11) + P(12) = 0.019 + 0.004 + 0.006 = 0.029.$$

Since this probability is less than 0.05, it would be considered unusual.

End of Lecture #10