MATH 108: Elementary Probability and Statistics

Ramapo College of New Jersey

Instructor: Dr. Atul Anurag

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Binomial Probability Distributions

Criteria for a Binomial Probability Experiment

An experiment is said to be a **binomial experiment** if it satisfies all of the following conditions:

1. The experiment is performed a fixed number of times. Each repetition is called a **trial**.
2. The trials are **independent**. That is, the outcome of one trial does not affect the outcome of any other trial.
3. Each trial results in one of two mutually exclusive outcomes: **success** or **failure**.
4. The probability of success, denoted by p, is the same for each trial.

Notation Used in the Binomial Distribution

- n: Number of independent trials.
- p: Probability of success on a single trial.
- $1 - p$: Probability of failure on a single trial.
- X: Random variable representing the number of successes in n trials.
  So the values of X range from 0 to n, i.e., $X \in \{0, 1, 2, \ldots, n\}$.

1. Determining Whether an Experiment is Binomial

A **binomial experiment** must meet the following four conditions:
Examples:

- Tossing a coin 5 times Binomial
- Rolling a die and recording each number Not Binomial
- Drawing cards without replacement Not Binomial (not independent)

Example Problem: Identifying Binomial Experiments

Determine which of the following probability experiments qualify as **binomial experiments**.
For those that are binomial experiments, identify:

- The number of trials n,
- The probability of success p,
- The probability of failure $q = 1 - p$,
- The possible values of the random variable X.

(a) Free Throws

A basketball player who historically makes 80% of her free throws is asked to shoot three free throws. The number of free throws made is recorded.

Solution:
This **is a binomial experiment**.

- Fixed number of trials: $n = 3$
- Independent shots (assumed): Yes
- Only two outcomes per shot: Made (success) or missed (failure)
- Constant probability of success: $p = 0.80$, so $q = 0.20$
- Random variable X: Number of shots made $X \in \{0, 1, 2, 3\}$

(b) Ice Cream Flavor

According to a recent Harris Poll, 28% of Americans state that chocolate is their favorite flavor of ice cream. A simple random sample of 10 people is selected, and the number who say chocolate is their favorite is recorded.

Solution:
This **is a binomial experiment**.

- Fixed number of trials: $n = 10$
- Independent responses (assumed): Yes
- Two outcomes: Likes chocolate (success) or not (failure)
- Constant probability: $p = 0.28$, $q = 0.72$
- Random variable X: Number who choose chocolate $X \in \{0, 1, \ldots, 10\}$

(c) Drawing Cards

Three cards are drawn from a standard deck **without replacement**, and the number of aces drawn is recorded.

Solution:
This **is not a binomial experiment**.

- Trials are **not independent** because cards are drawn without replacement.
- The probability of success changes from one trial to the next.

Therefore, the experiment violates the binomial conditions.

2. Computing Binomial Probabilities

The probability of getting exactly x successes in n trials is given by:

$$P(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

Where:

- $\binom{n}{x} = \dfrac{n!}{x!(n-x)!}$
- p: Probability of success
- $1 - p$: Probability of failure

Practice: Binomial Probability Calculations

In Problems 1720, a binomial probability experiment is conducted with the given parameters. Compute the probability of x successes in the n independent trials of the experiment.

Binomial Probability Formula:

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

17. $n = 10, \quad p = 0.4, \quad x = 3$
18. $n = 15, \quad p = 0.85, \quad x = 12$
19. $n = 40, \quad p = 0.99, \quad x = 38$
20. $n = 50, \quad p = 0.02, \quad x = 3$

Calculator Tip: Use a binomial probability function such as:

- `binompdf(n, p, x)` For exact value $P(X = x)$

Solutions

17. `binompdf(10, 0.4, 3)` $= \boxed{0.215}$
18. `binompdf(15, 0.85, 12)` $= \boxed{0.250}$
19. `binompdf(40, 0.99, 38)` $= \boxed{0.182}$
20. `binompdf(50, 0.02, 3)` $= \boxed{0.139}$

Example Problem: Binomial Probability Calculation

According to CTIA, 72% of all adult Americans would rather give up chocolate than their cell phone. In a random sample of 10 adult Americans, what is the probability that:

(a) Exactly 8 would rather give up chocolate?

(b) Fewer than 3 would rather give up chocolate?

(c) At least 3 would rather give up chocolate?

(d) The number of adult Americans who would rather give up chocolate is between 5 and 7, inclusive?

Given:

- Number of trials: $n = 10$
- Probability of success (give up chocolate): $p = 0.72$
- Probability of failure (would not give up chocolate): $q = 1 - p = 0.28$
- Random variable X: Number of people who would rather give up chocolate

Binomial Formula:

$$P(X = x) = \binom{n}{x} p^x q^{n-x}$$

Solution Outline:

(a) $P(X = 8) = \binom{10}{8}(0.72)^8(0.28)^2$

(b) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

(c) $P(X \geq 3) = 1 - P(X < 3) = 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$

(d) $P(5 \leq X \leq 7) = P(X = 5) + P(X = 6) + P(X = 7)$

Note: These probabilities can be calculated using a calculator with binomial functions or statistical software.
Example: A coin is flipped 4 times. What is the probability of getting exactly 2 heads?

$$n = 4, \quad x = 2, \quad p = 0.5$$

$$P(2) = \binom{4}{2}(0.5)^2(0.5)^2 = 6 \times 0.25 \times 0.25 = \boxed{0.375}$$

3. Mean and Standard Deviation of a Binomial Distribution

Formulas:

$$\mu = n \cdot p \qquad \text{(Mean)}$$
$$\sigma = \sqrt{n \cdot p \cdot (1 - p)} \qquad \text{(Standard Deviation)}$$

Example: A quiz has 10 true/false questions. If you guess on each question:

$$n = 10, \quad p = 0.5$$

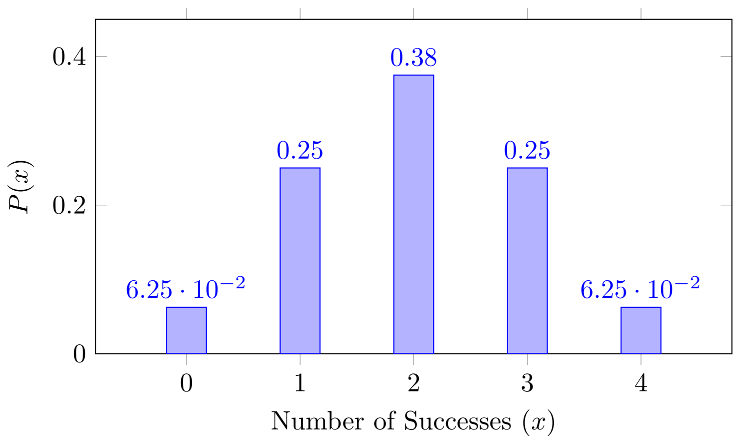
$$\mu = 10 \cdot 0.5 = \boxed{5} \qquad \sigma = \sqrt{10 \cdot 0.5 \cdot 0.5} = \sqrt{2.5} \approx \boxed{1.58}$$

Interpretation: On average, you'd get 5 questions right with a typical deviation of 1.58.

4. Graphing a Binomial Distribution

Use a bar graph to represent $P(x)$ for $x = 0$ to $x = n$.
Example: Flip a fair coin 4 times ($n = 4, p = 0.5$):

x	$P(x)$
0	0.0625
1	0.25
2	0.375
3	0.25
4	0.0625



Shape: Symmetric for $p = 0.5$. Becomes skewed as p deviates from 0.5.

Technology Tip: Using Calculator or Software

Many calculators (e.g., TI-84) and software like Excel or Python can compute binomial probabilities.
TI-84:

- binompdf(n, p, x) for exact value
- binomcdf(n, p, x) for cumulative probability

Excel:

`=BINOM.DIST(x, n, p, FALSE)` for exact`=BINOM.DIST(x, n, p, TRUE)` for cumulative

Binomial Distribution Practice: Problems 2932

In Problems 2932:

(a) Construct a binomial probability distribution with the given parameters.

(b) Compute the mean and standard deviation of the random variable using methods from Section 6.1 (e.g., using the full distribution).

(c) Compute the mean and standard deviation using the shortcut formulas:

$$\mu = np \quad \text{and} \quad \sigma = \sqrt{np(1-p)}$$

(d) Draw a graph of the probability distribution and comment on its shape.

29. $n = 6, \quad p = 0.3$
30. $n = 8, \quad p = 0.5$
31. $n = 9, \quad p = 0.75$
32. $n = 10, \quad p = 0.2$

Example Solution for Problem 29: $n = 6, p = 0.3$

(a) Table below:

x	$P(X = x)$
0	0.1176
1	0.3025
2	0.3241
3	0.1852
4	0.0595
5	0.0102
6	0.0007

(b) Compute mean and standard deviation using weighted sums:

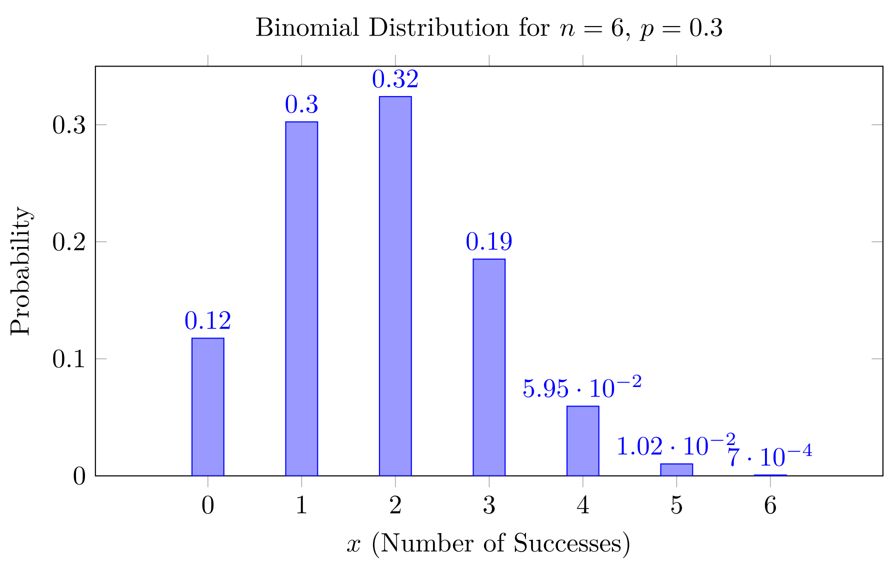
$$\mu = \sum x \cdot P(X = x), \quad \sigma = \sqrt{\sum (x - \mu)^2 \cdot P(X = x)}$$

(c) Shortcut method:

$$\mu = np = 6 \cdot 0.3 = 1.8 \quad \text{and} \quad \sigma = \sqrt{6 \cdot 0.3 \cdot 0.7} \approx 1.122$$

(d) **Shape:** Skewed right (since $p < 0.5$). Distribution is concentrated toward lower values.

Graph for Problem 29: $n = 6, \ p = 0.3$



Comment on Shape: The distribution is *right-skewed* because $p = 0.3 < 0.5$. Most probability mass is concentrated on lower values of x.

End of Lecture #11