MATH 108: Elementary Probability and Statistics

Ramapo College of New Jersey

Instructor: Dr. Atul Anurag

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Probability

1. Basic Definitions

Sample Space (S):

The set of all possible outcomes of a random experiment.

Example:

Rolling a die

$$S = \{1, 2, 3, 4, 5, 6\}$$

Event (E):

An event is any subset of the sample space.

Example:

Getting an even number on a die roll

$$E = \{2, 4, 6\}$$

2. Complement of an Event

Definition:

The complement of an event E, denoted by E^c, is the set of all outcomes in the sample space that are NOT in E.

Important Identity:

$$P(E) + P(E^c) = 1$$

This means: The probability of an event happening plus the probability of it not happening is always 1.

Example: Rolling a Die

Let's say we roll a fair 6-sided die.

- Sample space: $S = \{1, 2, 3, 4, 5, 6\}$
- Let E be the event "rolling an even number": $E = \{2, 4, 6\}$
- Complement E': "rolling an odd number": $E' = \{1, 3, 5\}$

$$P(E) = \frac{3}{6} = 0.5, \quad P(E') = \frac{3}{6} = 0.5$$
$$P(E) + P(E') = 0.5 + 0.5 = 1$$

Why Complements Are Useful

It is often easier to calculate the probability of the complement and subtract from 1:

$$P(E) = 1 - P(E^c)$$

Example: At least one heads in 2 coin tosses

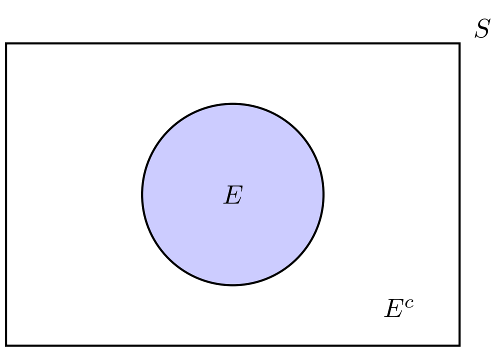
- Sample space: $S = \{HH, HT, TH, TT\}$
- Event E: at least one heads
$$E = \{HH, HT, TH\}$$
- Complement E^c: no heads (all tails)
$$E^c = \{TT\}$$

$$P(E^c) = \frac{1}{4}, \quad P(E) = 1 - \frac{1}{4} = \frac{3}{4}$$

5. Venn Diagram Representation

- Sample space S is a rectangle.
- Event E is a circle inside it.
- Area outside the circle but inside the rectangle represents E^c.

$$P(E) + P(E^c) = 1, \quad E \cap E^c = \emptyset \text{ (empty set)}, \quad E \cup E^c = S$$



- $P(E) + P(E^c) = 1$
  - The probability of an event E plus the probability of its complement E^c is always 1.
  - This means either the event happens, or it doesn't—no other options exist.
  - Based on the total probability rule: the probability of the entire sample space is 1.

- $E \cap E^c = \emptyset$
  - The intersection of an event and its complement is the empty set: E and E^c have no outcomes in common.
  - They are mutually exclusive—if one occurs, the other cannot.
- $E \cup E^c = S$
  - The union of an event and its complement equals the entire sample space S.
  - Every possible outcome is either in E or in E^c.
  - Together, they account for all outcomes in the sample space.

Example: Flipping a Coin 3 Times (9-Part Breakdown)

Event 1. Sample Space:

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Event 2. Total Outcomes in the sample space: 8

Event 3. Event A: All heads

$$A = \{HHH\}$$
$$P(A) = \frac{1}{8}$$

Event 4. Event B: At least one tail

$$B = \{HHT, HTH, HTT, THH, THT, TTH, TTT\}$$
$$P(B) = \frac{7}{8}$$

Event 5. Complement of B: No tails (i.e., all heads)

$$B^c = \{HHH\}$$
$$P(B^c) = \frac{1}{8}$$

Event 6. Event C: Exactly two heads

$$C = \{HHT, HTH, THH\}$$
$$P(C) = \frac{3}{8}$$

Event 7. Using the Complement Rule:

$$P(B) = 1 - P(B^c) = 1 - \frac{1}{8} = \frac{7}{8}$$

Event 8. Summary Table:

Event	Description	Outcomes	Probability
A	All heads	HHH	$\frac{1}{8}$
B	At least one tail	All except HHH	$\frac{7}{8}$
B^c	No tails	HHH	$\frac{1}{8}$
C	Exactly two heads	HHT, HTH, THH	$\frac{3}{8}$

7. Question: Rolling a Die Twice

There are $6 \times 6 = 36$ possible outcomes when two fair dice are rolled. Each outcome is represented as an ordered pair (x, y), where x is the result from the first die and y from the second. The sum is $x + y$.

First \ Second	**1**	**2**	**3**	**4**	**5**	**6**
1	(1,1)=2	(1,2)=3	(1,3)=4	(1,4)=5	(1,5)=6	(1,6)=7
2	(2,1)=3	(2,2)=4	(2,3)=5	(2,4)=6	(2,5)=7	(2,6)=8
3	(3,1)=4	(3,2)=5	(3,3)=6	(3,4)=7	(3,5)=8	(3,6)=9
4	(4,1)=5	(4,2)=6	(4,3)=7	(4,4)=8	(4,5)=9	(4,6)=10
5	(5,1)=6	(5,2)=7	(5,3)=8	(5,4)=9	(5,5)=10	(5,6)=11
6	(6,1)=7	(6,2)=8	(6,3)=9	(6,4)=10	(6,5)=11	(6,6)=12

Event 1. Sum is 7:

$$A = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$$

$$P(A) = \frac{6}{36} = \frac{1}{6}$$

Event 2. Doubles (both dice show the same number):

$$B = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$$

$$P(B) = \frac{6}{36} = \frac{1}{6}$$

Event 3. First die is 4:

$$C = \{(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\}$$

$$P(C) = \frac{6}{36} = \frac{1}{6}$$

Event 4. Complement of C (first die is not 4):

$$P(C^c) = 1 - \frac{6}{36} = \frac{30}{36}$$

Event 5. Sum is less than or equal to 4:

$$D = \{(1,1),(1,2),(1,3),(2,1),(2,2),(3,1)\}$$

$$P(D) = \frac{6}{36} = \frac{1}{6}$$

Event 6. At least one die shows a 6:

$$E = \{(1,6),(2,6),(3,6),(4,6),(5,6),(6,6),(6,1),(6,2),(6,3),(6,4),(6,5)\}$$

$$P(E) = \frac{11}{36}$$

Event 7. Sum is even:

18 outcomes have even sums

$$P(F) = \frac{18}{36} = \frac{1}{2}$$

Event 8. Complement of F (sum is odd):

$$P(F^c) = 1 - \frac{1}{2} = \frac{1}{2}$$

Event 9. Not a sum of 7:

$$P(G) = 1 - \frac{6}{36} = \frac{30}{36} = \frac{5}{6}$$

End of Lecture #7